Attitude Control and Determination System *for* **DTUsat1**

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Abstract

Different methods of attitude control and determination are considered and measured against the physical constraints, feasability and performance.

Solar sensors and magnetic field sensors are considered the most realistic attitude determination sensors. The requirement to the attitude control can be either single axis or three axes control. If single axis control is required a solution with an actively damped magnet or magnetotorquers are suggested. If three axes control is required a magnetotorquer with three or six coils is suggested.

Contents

Chapter 1

Introduction

Several subsystems in a satellite may require a stable satellite for better performance or to function at all. For instance radio communication will require less power if the antenna(s) can be pointed towards the Earth and the solar panels may increase power output if properly directed towards the Sun. Several of the discussed payloads also requires a stabilized platform where pitch, roll and yaw are controlled.

The above described requirements states that an attitude control system is needed. This report describes the relevant attitude control and determination strategies available for small satellites. Furthermore it states which strategies that can be utilised for implementation on **DTUsat1** when the available amount of time, man power, knowledge, money and possible payloads is taken into consideration.

Chapter 2

Problem statement

2.1 Scope

The attitude control and determination system (ACDS) has the responsibility for:

- *Initial Acquisition* – detumbling of the satellite which brings the attitude of the satellite into a predefined set of conditions. The initial acquisition can also be used as a safe mode[1] function.
- *Operation* – more precise control of the satellite's attitude. The requirements for this function are payload and system dependent.

The operation function of the ACDS may also be divided into sub-functions:

- *Attitude Determination Sensor(s).*
- *Attitude Control Actuator(s).*
- *Data Handling Unit*[2] for processing sensor data and control of actuators.

Requirements for interfacing other sub-systems may include:

- *Solar Panels Interface* – access to charge current and preferably also temperatures, allowing the solar panels to be utilized as Sun sensor.
- *Radio Interface* – access to wide and narrow band signal strength; enabling the radio to be used respectively for ground detection and fine adjustment of the bearing towards the ground station.

[1] The safe mode of a satellite is a mode where only critical subsystems are operating with the purpose of allowing communication with the ground station.

[2] Most likely a task on the main computer.

Additional interfaces may include interfacing with internal sensors and actuators within the ACDS.

Chapter 3

Technologies

3.1 Attitude Determination

Determination of a spacecraft's orientation is necessary when the attitude is to be controlled actively. In the following a number of different sensors will be evaluated and compared to the requirements and constrains of the **DTUsat1** mission. Attitude sensors can be divided into two categories namely reference and inertial sensors. The reference sensors determines the attitude based on fixed objects like the Sun or stars, but - depending on the orbit - these objects can occasionally be eclipsed. Therefore it is necessary to monitor the crafts motion during an eclipse hence using an inertial sensor. The latter senor is not able to provide the attitude without being calibrated by a reference sensor. The use of an inertial sensor can be avoided by having several reference sensors using different fixed objects.

3.1.1 Sun Sensor

As the word implies this sensor gives a direction to the sun. Depending on the quality of the sensor the output can vary from a general direction to a 1 arc minute precise vector. The latter is calculated by having a slit in front of the detector element, and thereby making it possible to determine the angle of the sunrays.

Feasibility

A realistic sun sensor is in our case to measure the charge current from the individual solar panels or a dedicated single element solar sensor. The latter could be a light sensitive diode or a sensor as described above which could be made relatively small.

3.1.2 Earth Sensor

Two approaches are generally used, ground or horizon detection. Ground detection can determine if the Earth is positioned in the field of view by measuring a unique Earth signature. Horizon detection extends the previous by detecting the transition between Earth and space. Having several horizon fixpoints enables calculation of the Earth's center.

Feasibility

Ground detection can be realised by measuring the temperature by means of infrared light, or measuring the broadband signal strength with a radio, selecting a radio segment with high probability of earth based transmissions eg. the FM band.

Detecting multiple horizon points almost simultaneously is somewhat unrealistic in our case, due to complexibility and computational load. Existing commercial solutions are to large for a **CubeSat**.

3.1.3 Star Sensor

This normally consists of a CCD camera, a quartz lens and a baffle. This type of sensors requires a lot of computation and memory, since a star catalog must be compared to the picture taken by the CCD camera approximately each second. The axes of the camera can be calculated and the attitude determined with an accuracy of less than 1 arc sec.

Feasibility

Due to size and weight alone it is unrealistic to implement a star sensor.

3.1.4 Magnetic Field Sensor

This sensor has not found much use in the history of satellites due to lag of precise mapping of the Earth's magnetic field; especially field abnormalities. But since the newly obtained data from Ørsted and increased development of magnetic field sensors, more precise results can be found. A magnetic field sensor measures the local field's strength and direction. This sensor is necessary e.g. when the ACS is based on magnetotorquers. The position accuracy is 30 arc minutes, but this value is six years old so the actual value is more in the vicinity of 5 arc minutes given that the accuracy of the sensor is high enough.

Feasibility

A small enough solution is using a Hall sensor for each direction to make an absolute measurement or by generating a local field and detect the cancellation of the Earth's field. The attitude is determined by comparing the found field vector with a lookup table.

Complete three axis sensors with build-in cancellation coils are available in integrated versions.

Due to limited dynamic range in the sensor circuit the use of permanent magnets and magnetic field sensors becomes difficult or even impossible.

3.1.5 Gyroscope

Gyroscopes are inertial sensors and are able to measure angular velocity. There exists a wide range of gyros from the very expensive laser gyros that are used on Ariane 4 to the small piezo-electric vibratory gyros.

Feasibility

A further study of piezo-electric gyros is needed concerning long time drift, temperature stability and liability.

3.2 Attitude Control

There are several possible ways to achieve attitude control, in the following different methods are discussed.

3.2.1 Fly Wheels

By spinning up and down a fly wheel, it is possible to generate an angular momentum. In the three dimensional case three fly wheels are fitted pointing in perpendicular directions. If redundancy is required four fly wheels are placed in the corners of a three sided pyramid.

Feasibility

Friction between moving mechanical parts in space requires special care in selecting materials. It is also necessary to consider means of dissipating angular momentum due to limitation in angular velocity of the flywheels.

If fly wheels are to be used they are required to be capable of delivering higher angular momentum than the spinning satellite[1]. In order to find fly wheel candidates the moment of inertia is needed since $\boldsymbol{L} = I\boldsymbol{\omega}$. The moment of inertia for the satellite structure was estimated with a MatLab script to the values listed in table 3.1.

I_x	$6.858 \cdot 10^{-4}\text{kgm}^2$
I_y	$6.858 \cdot 10^{-4}\text{kgm}^2$
I_z	$8.164 \cdot 10^{-4}\text{kgm}^2$

Table 3.1: Estimated moment of inertia for the structure.

By using the estimated I_z and the structure mass on approximately 230g the structure can be equivalised with a worst case ball shell with radius:

$$I_{shell} = \frac{2}{3} M_{structure} R^2 \Leftrightarrow R = \sqrt{\frac{3I_{shell}}{2M_{structure}}}$$
$$= \sqrt{\frac{3 \cdot 8.1642 \cdot 10^{-4}\text{kgm}^2}{2 \cdot 0.2300\text{kg}}} = 7.755 \cdot 10^{-2}\text{m}$$

The **DTUsat1** (worst case mass: 1kg) is now equivalised with a worst case ball where 40% of the mass is located on the shell and the rest evenly dis-

[1] It you do not want to dissipate angular momentum during the detumble.

tributed within the ball. This yields the below given moment of inertia.

$$\begin{aligned} I = I_{shell} + I_{interior} &= \frac{2}{3}M_{40\%}R^2 + \frac{2}{5}M_{60\%}R^2 \\ &= \left(\frac{4}{15} + \frac{6}{25}\right)MR^2 = \frac{38}{75}MR^2 \\ &= \frac{38}{75} \cdot 1.00\text{kg} \cdot (7.755 \cdot 10^{-2}\text{m})^2 = 3.05 \cdot 10^{-3}\text{kgm}^2 \end{aligned}$$

When a **CubeSat** is released from the P-POD it is expected to have a maximum tumble of 2 rounds per minute. In order to be completely certain that the **DTUsat1** can be stabilized it has been decided to make it capable of handling a tumble on 5rpm. This means that a fly wheel must be capable of delivering the below calculated angular momentum.

$$\boldsymbol{L_{wc}} = I\boldsymbol{\omega} = 3.05 \cdot 10^{-3}\text{kgm}^2 \cdot 2\pi \cdot \frac{5}{60}\text{s}^{-1} = 1.60 \cdot 10^{-3}\frac{\text{kgm}^2}{\text{s}}$$

Example with Maxon EC-32 motor

This motor can detumble the **DTUsat1** at n RPM (without any extra momentum disks attached):

$$\begin{aligned} & I_M\boldsymbol{\omega_M} = L_{wc} \Leftrightarrow \boldsymbol{\omega_M} = \frac{L_{wc}}{I_M} \\ & \Rightarrow n = \frac{\boldsymbol{\omega_M}}{2\pi} \cdot 60\text{s} = \frac{60\text{s}L_{wc}}{2\pi I_M} = \frac{60\text{s} \cdot 1.60 \cdot 10^{-3}\frac{\text{kgm}^2}{\text{s}}}{2\pi 13.9 \cdot 10^{-7}\text{kgm}^2} = 10971\text{RPM} \end{aligned}$$

This shows that if this motor is to be used it must be loaded with extra momentum disks since the motor has a maximum no load speed on 8600RPM (at max 3W). However it can be used without momentum disks if the angular momentum can be dissipated during the detumbling process.

If it is desired to lower the fly wheel's rotation speed then the rotational energy must be unloaded to the magnetic field with magnetotorquers. The rotational energy can at low power be unloaded with a 1μNm torque. Spinning down the motor 1000RPMs takes:

$$\begin{aligned} & \frac{\text{d}\boldsymbol{L}}{\text{d}t} = \boldsymbol{\tau} = constant \Rightarrow \\ & \frac{\Delta\boldsymbol{L}}{\Delta t} = \boldsymbol{\tau} \Leftrightarrow \\ & \Delta t = \frac{\Delta\boldsymbol{L}}{\boldsymbol{\tau}} = \frac{I\Delta\boldsymbol{\omega}}{\boldsymbol{\tau}} = \frac{3.05 \cdot 10^{-3}\text{kgm}^2 \cdot 2\pi \cdot \frac{1000}{60\text{s}}}{1\mu\text{Nm}} = 88.72\text{hours} \end{aligned}$$

3.2.2 Magnetotorquers

When inducing a current in an inductor a magnetic field is generated. The generated field will align with the Earth's magnetic field and hereby turning the satellite. The control is limited to certain places in the Earth's magnetic field.

Feasibility

The control algorithms will be more complicated since maneuvers only can be made at certain points in the orbit.

A three axes design example gave a coil mass of 100g and an estimated 1.75μNm torque per 50mW.

It is necessary to have a three axes measurement of the Earth's magnetic field, with a reasonable (need to be investigated more) accuracy.

3.2.3 Passive Magnetic Control

A magnet is placed inside the satellite aligning the satellite with the magnetic field of the Earth.

Feasibility

Easy and foolproof. This will give an undamped but stable control; the system can be damped either passively or actively. Passive damping can be made with threads of magnetic material with large hysteresis or active with small magnetotorquers.

3.2.4 Gravity Control

By distributing the mass for obtaining two mass system equivalency the gravity gradient can be used to align the satellite perpendicular to the surface of the Earth.

This method has two stable and one meta stable state - the stable states are the above mentioned and the meta stable state is perpendicular to these.

Feasibility

Gravity control can be implemented by fitting a counterweight to the satellite at the end of a long boom or string.

The probability of remaining in the meta stable state is small, because of the disturbance from e.g. drag forces.

From a mechanical point of view this solution is very difficult since a **CubeSat** must fulfill certain dimension constraints.

Chapter 4

Design Options

4.1 Payload Dependency

The different payloads has different requirements to the ACDS system. At present four different payload suggestions remain.

4.1.1 Tether System

This payload requires a precise knowledge of the attitude and a slow varying attitude in the tether deployment phase. Three axes control is at the moment not a requirement, but further study may yield a different result. After deployment it may be necessary that the satellite does not spin – again this requires further study from the payload group.

4.1.2 Active Pixel Sensor (Camera)

At present we do not know the shutter time for a APS. If this is low a high performance 3 axis control system is necessary to avoid blurred pictures; especially if the camera takes pictures of distant objects - eg. the Earth.

This means that if the shutter time is short, fly wheels are the only attitude control system that are able to react fast enough to be feasible.

At present a requirement of 0.1rad/sec is considered reasonable according to Johnny Hartvig Olsen from the Payload group, but if the camera needs an exposure time on 1sec it will require a stability on $1 \cdot 10^{-4}$rad/sec to take a picture of the Earth (also acording to JHO). The calculations are not yet final.

	Weight.	Pas. Damped Mag.	Act. Damped Mag.	M.torquers	Fly Wheels
Mass	1	++	+	0	-
Size	1	0	+	-	--
Power	2	++	+	+	-
Precision	2	0	0	+	++
Man Power	3	--	0	0	+
Feasibility	2	0	++	+	--
Construction Risk	2	--	-	0	+
Operation Risk	2	++	+	0	-
Response Time	1	--	-	0	++
Magnetometer	2	--	--	+	-
Result		-6	3	7	-2

Table 4.1: Trade off between the different Attitude Control designs.

4.1.3 Geiger Tube

No attitude control is necessary, but if present the Geiger tube will be able to measure the direction of radiation. This puts a requirement on the attitude determination.

4.1.4 Deployable Solar Panels

To gain benefit from deployable solar panels it is necessary to point the panels towards the Sun. Full 3 axes control is therefore necessary. Magnetotorquers and fly wheels are the possible solutions.

4.2 Trade Off between Different Technologies

Weight Assignment

The total mass, size, power etc. of the attitude control and determinination hardware system, excluding onboard computer, is weighted from a nominally selected level.

Mass – 100g

Size – 10cm^3

Power – 100mW

Precision – 10°

Man Power – 60 ECTS points, approx. 1000 man hours.

Feasibility – Subjective evaluation of complexity and accesibility to materials.

Construction Risk – Risks due to faulty calculations and material data.

Operation Risk – Risk during operation due to fault in moving mechanical parts and electrical circuits.

Response Time – Expected control speed normalised around the magnetotorquer solution.

Magnetometer – The posibility to implement a magnetometer in conjunction with the attitude control system is weighted positive since this is the most realistic attitude determination.

The rows in table 4.2 are weighted according to the expected limitations in the subsystem, compared to the above nominal levels.

The mass, size and power budgets are not yet determined. We expect to have access to 200g, 100cm^3 and 200mW which in turn means that our nominal levels are low compared to the budget, the weight for these are therefore low to medium.

Precision is given a medium weight but this is very payload dependent and may change after final payload selection. Without payload requirements we would still like to demonstrate the ability to control the attitude of the satellite.

The man power necessary to complete the task of designing, prototyping and testing the ACDS must be within the given resources, so this is almost considered to be a constraint – thus it has been assigned a heigh weight.

The importance of responce time is low considering possible payloads.

The ability to have a magnetometer onboard is assigned a medium weight. Onboard permanent magnets require high dynamic range of the magnetometer or the posibillity of using mean measurement if the magnets are rotating as with fly wheels.

Trade Off Conclusion

If the payload requires three axes control the trade off study shows an obvious advantage to magnetotorquers over fly wheels, mainly due to physical constraints and feasibility.

If only single axis control is required, the trade off study gives the option of a permanent magnet actively damped by weak two axes magnetotorquers. If the north and south pole tumbling is inacceptible a solution with magnetotorquers is useable.

Chapter 5

ACDS Overview

5.1 Resources

- *Man Power* – approximately 100ECTS points.
- *Hardware* – needs further investigation.
- *Software Tools* – Nova[1], Spenvis and MatLab.
- *Facilities* – depends on final design choice; however it does not seem like special (expensive) equipment is necessary.
- *Costs* – fly wheel motors and magnetic field sensors are expected to be in the range of 1.000DKK-4.000DKK. It has not been possible to obtain exact figures on expected costs; however total costs are estimated to be well within 40.000DKK (worst case).

5.2 Make or Buy Recommendations

Most of the ACDS will be developed by the ACDS group. It has however been considered to buy the following items (it they are needed):

- Magnetic field sensor chip.
- Fly wheel motors.

since these functions can be big projects on their own.

[1] The evaluation version is probably sufficient.

5.3 Requirements Capture

To build a sufficient ACDS the following requirements must be fulfilled.

- Access to charge current from the solar panels.
- Access to wide and narrow band signal strength.
- Center of mass must be as close to the geometric center as possible – hence placeing the relative heavy components accordingly. This requirement will be further specified on a system engineering meeting.
- Room for coils – possibly imbedded in side panels.

5.4 Risk Factors

Major critical risk factors are

- Design
- Electrical
- Mechanical failures.

The risk factors depends on the design – one design option include moveable parts (fly wheels) and all but one (passive magnetic control) include electrical parts.

5.5 Recommendation

5.5.1 Single axis stabilization

Considering risk as the primary factor a permanent magnet with active damping is the most attractive solution. The calculation and theory needed to design the passive damping is not realistic as the knowledge is not present at DTU.

A single axis magnetotorquers solution can prevent the tumbling of the satellite when passing the magnetic poles. This allows a fixed rotation adapted to the Earth's rotation.

5.5.2 Three axis stabilization

Magnetotorquers require a precise measurement and map of the Earth's magnetic field. The control is limited to two axes over the poles and two axes over Equator. The control is slow because of the weak strength of the magnetic field.

Fly wheels require a way to dump angular momentum to keep the wheels below maximum speed. This is done by using (weak) magnetotorquers. It is not necessary to use a map of the Earth's magnetic field, since it is sufficient to know the present direction.

A decision between magnetotorquers and fly wheels requires a final payload decision and further investigation. If fast control is a vital requirement fly wheels is the solution.

5.6 Criteria of Success

Ordered list of success levels:

- Receiving attitude sensor information.
- Detumbling the satellite (initial acquisition).
- Bringing the satellite into an attitude that allows quality radio communication.
- Fulfilling payload's attitude control requirements.
- Successful attitude control by means of radio signal strength.

Appendix A

Magnetotorquers

A.1 Basic magnetostatics

The magnetic force $\boldsymbol{F_m}$ on a charge q moving with the velocity $\boldsymbol{v}$ through a homogeneous magnetic field $\boldsymbol{B}$ is given by (A.1).

$$\boldsymbol{F_m} = q\boldsymbol{v} \times \boldsymbol{B} \tag{A.1}$$

For a wire of length l with current $\boldsymbol{I}$ this translates into (A.2).

$$\begin{aligned} q\boldsymbol{v} &= l\boldsymbol{I} \\ \Rightarrow \boldsymbol{F_m} &= l\boldsymbol{I} \times \boldsymbol{B} \end{aligned} \tag{A.2}$$

This force will in some situations create a torque around a rotation point – for a free floating body this will be the center of mass. The torque $\boldsymbol{\tau}$ is given by the vector $\boldsymbol{R}$ from the rotation point to the point of the applied force and the applyed force $\boldsymbol{F}$ as shown in (A.3).

$$\boldsymbol{\tau} = \boldsymbol{R} \times \boldsymbol{F} \tag{A.3}$$

A.2 Basic magnetotorquer

By nature all currents flow in closed loops and not in single straight wires. The basic model for magnetotorquers therefore consists of a single current loop as shown in figure A.1.

In the following equations the forces $\boldsymbol{F}_i$, torques $\boldsymbol{\tau}_i$ and the magnetic field $\boldsymbol{B}$ are projected on to the x, y and z axes where i indicates that the forces and torques origin from wire $\textcircled{i}$.

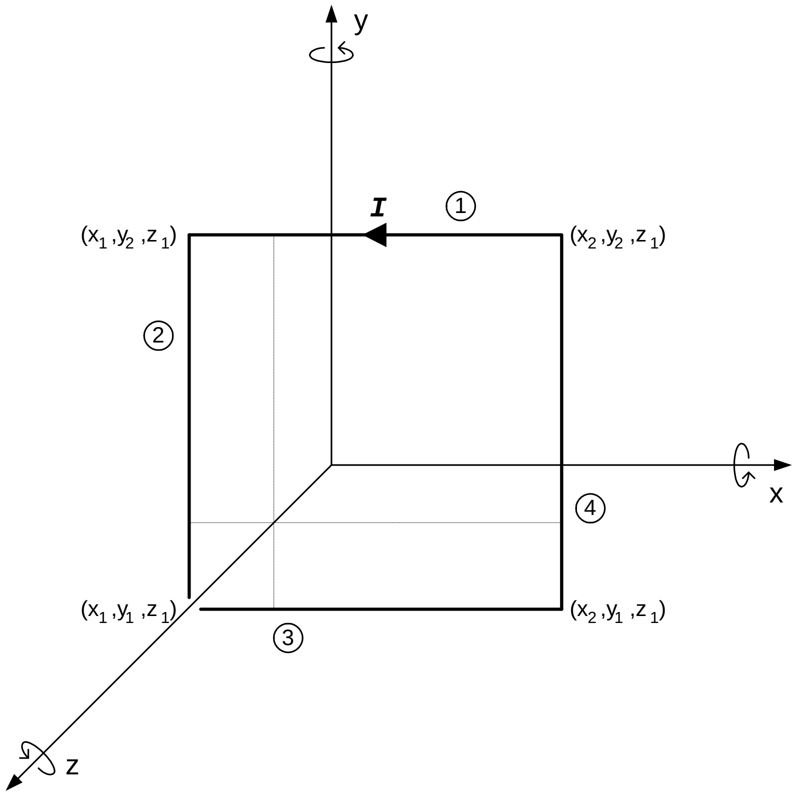

Figure A.1: Outline of basic magnetotorquer.

$$
\begin{aligned}
F_{1\ x} &= 0 \\
F_{1\ y} &= I(x_2 - x_1)B_z \\
F_{1\ z} &= -I(x_2 - x_1)B_y \\
\tau_{1\ x} &= -z_1 F_{1\ y} + y_2 F_{1\ z} \\
&= -I(x_2 - x_1)(z_1 B_z + y_2 B_y) \\
\tau_{1\ y} &= -\int_{x_1}^{x_2} \frac{F_{1\ z}}{x_2 - x_1} x \mathrm{d}x \\
&= \frac{1}{2} I B_y ({x_2}^2 - {x_1}^2) \\
\tau_{1\ z} &= \int_{x_1}^{x_2} \frac{F_{1\ y}}{x_2 - x_1} x \mathrm{d}x \\
&= \frac{1}{2} I B_z ({x_2}^2 - {x_1}^2)
\end{aligned}
$$

$$
\begin{aligned}
F_{2\ x} &= -I(y_2 - y_1)B_z \\
F_{2\ y} &= 0 \\
F_{2\ z} &= I(y_2 - y_1)B_x \\
\tau_{2\ x} &= \int_{y_1}^{y_2} \frac{F_{2\ z}}{y_2 - y_1} y \mathrm{d}y \\
&= \frac{1}{2} I B_x ({y_2}^2 - {y_1}^2) \\
\tau_{2\ y} &= -x_1 F_{2\ z} + z_1 F_{2\ x} \\
&= -I(y_2 - y_1)(x_1 B_x + z_1 B_z) \\
\tau_{2\ z} &= -\int_{y_1}^{y_2} \frac{F_{2\ x}}{y_2 - y_1} y \mathrm{d}y \\
&= \frac{1}{2} I B_z ({y_2}^2 - {y_1}^2)
\end{aligned}
$$

$$
\begin{aligned}
F_{3\ x} &= 0 \\
F_{3\ y} &= -I(x_2 - x_1)B_z = -F_{1\ y} \\
F_{3\ z} &= I(x_2 - x_1)B_y = -F_{1\ z} \\
\tau_{3\ x} &= -z_1 F_{3\ y} + y_1 F_{3\ z} \\
&= I(x_2 - x_1)(z_1 B_z + y_1 B_y) \\
\tau_{3\ y} &= -\int_{x_1}^{x_2} \frac{F_{3\ z}}{x_2 - x_1} x \mathrm{d}x \\
&= -\frac{1}{2} I B_y ({x_2}^2 - {x_1}^2) = -\tau_{1\ y} \\
\tau_{3\ z} &= \int_{x_1}^{x_2} \frac{F_{3\ y}}{x_2 - x_1} x \mathrm{d}x \\
&= -\frac{1}{2} I B_z ({x_2}^2 - {x_1}^2) = -\tau_{1\ z}
\end{aligned}
$$

$$
\begin{aligned}
F_{4\ x} &= I(y_2 - y_1)B_z = -F_{2\ x} \\
F_{4\ y} &= 0 \\
F_{4\ z} &= -I(y_2 - y_1)B_x = -F_{2\ z} \\
\tau_{4\ x} &= \int_{y_1}^{y_2} \frac{F_{4\ z}}{y_2 - y_1} y \mathrm{d}y \\
&= -\frac{1}{2} I B_x ({y_2}^2 - {y_1}^2) = -\tau_{2\ x} \\
\tau_{4\ y} &= -x_2 F_{4\ z} + z_1 F_{4\ x} \\
&= I(y_2 - y_1)(x_2 B_x + z_1 B_z) \\
\tau_{4\ z} &= -\int_{y_1}^{y_2} \frac{F_{4\ x}}{y_2 - y_1} y \mathrm{d}y \\
&= -\frac{1}{2} I B_z ({y_2}^2 - {y_1}^2) = -\tau_{2\ z}
\end{aligned}
$$

The total torques projected on the three axes can now be calculated:

$$
\begin{aligned}
\tau_x &= -I(x_2 - x_1)(z_1 B_z + y_2 B_y) \\
&+ \frac{1}{2} I B_x ({y_2}^2 - {y_1}^2) \\
&+ I(x_2 - x_1)(z_1 B_z + y_1 B_y) \\
&- \frac{1}{2} I B_x ({y_2}^2 - {y_1}^2) \\
&= -I(x_2 - x_1)(y_2 - y_1) B_y \\
&= -AIB_y \\
\tau_y &= \frac{1}{2} I B_y ({x_2}^2 - {x_1}^2) \\
&- I(y_2 - y_1)(x_1 B_x + z_1 B_z) \\
&- \frac{1}{2} I B_y ({x_2}^2 - {x_1}^2) = -\tau_{1\ y} \\
&+ I(y_2 - y_1)(x_2 B_x + z_1 B_z) \\
&= I(x_2 - x_1)(y_2 - y_1) B_x \\
&= AIB_x \\
\tau_z &= \frac{1}{2} I B_z ({x_2}^2 - {x_1}^2) \\
&+ \frac{1}{2} I B_z ({y_2}^2 - {y_1}^2) \\
&- \frac{1}{2} I B_z ({x_2}^2 - {x_1}^2) \\
&- \frac{1}{2} I B_z ({y_2}^2 - {y_1}^2) \\
&= 0
\end{aligned}
$$

where $A = (x_2 - x_1)(y_2 - y_1)$ is the cross area of the loop.
It is here by shown that the torques are independent of the loop relative to the rotation point since there is no dependency on the specific coordinates x_1, x_2, y_1, y_2 and z_1.

The torque can also be expressed using vector notation which uses the magnetic dipole moment $\boldsymbol{m}$ of the loop given by the current I, the surface S spanned by I and the vector $\boldsymbol{n}$ perpendicular to the surface and positive direction relative to I in agreement with the right hand rule.

$$
\begin{aligned}
&\boldsymbol{m} = \boldsymbol{n} IS \\
\Rightarrow\ &\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B}
\end{aligned}
$$

A.3 Implementing magnetotorquers

In order to maximize the torque the cross area and the current has to be maximized.

- Due to mechanical constraints an estimate of the realiseable cross area is 7cm by 7cm enabling a coil cross area of 5mm by 2.5mm.

- The fixed battery voltage and the limited available power should be used as design parameters for the coil in order to reduce power loss – e.g. by making step-up converters unnecessary.

With a design goal of 50mW per coil from a 4.8V battery the resistance becomes 460Ω which can only be achieved by making several turns. By doing this and using round enamelled wire one has to take the fill factor in to account when calculating the effective cross area of the coil.

The ideal fill factor for solid wires without enamel is approx. 90.7% and the fill factor for e.g. a 0.22 mm wire is approx. 66.9% which results in a total fill factor of approx. 60.7%. Because of the non-ideal winding of the coil a fill factor of 40% should be realiseable giving an effective coil cross area of $A_{eff.coil} = 5.00 \cdot 10^{-6}\text{m}^2$.

The total resistance of the wire R_{tot} is given by the specific resistance of copper r_s, the number of turns n, the width of the coil x_t, the height of the coil y_t and the cross area of the wire A_{wire}:

$$R_{tot} = r_s \frac{n(x_t + y_t)2}{A_{wire}} \tag{A.4}$$

The coil current I is then given by the battery voltage U_{bat} and the total resistance R_{tot}:

$$I = \frac{U_{bat}}{R_{tot}}$$

The total current through the coil's cross area I_{tot} is given by the number of turns n and the coil current I:

$$I_{tot} = nI$$

The coil current I can now be found:

$$
\begin{aligned}
P &= U_{bat} I = R_{tot} I^2 \\
&= r_s \frac{n(x_t + y_t)2}{A_{wire}} \left(\frac{I_{tot}}{n} \right)^2 \\
&= r_s \frac{2(x_t + y_t)}{A_{wire} n} {I_{tot}}^2 \\
\Leftrightarrow {I_{tot}}^2 &= \frac{P A_{wire} n}{r_s 2(x_t + y_t)} \\
\Leftrightarrow I_{tot} &= \sqrt{\frac{P A_{wire} n}{r_s 2(x_t + y_t)}}
\end{aligned}
$$

For $P = 50\text{mW}$, $A_{wire} n = A_{eff.coil} = 5.00 \cdot 10^{-6}\text{m}^2$, $r_s = 1.724 \cdot 10^{-8}\Omega\text{m}$ and $x_t = y_t = 0.07\text{m}$:

$$
I_{tot} = \sqrt{\frac{50\text{mW} \cdot 5.00 \cdot 10^{-6}\text{m}^2}{1.724 \cdot 10^{-8}\Omega\text{m} \cdot 2\,(0.07\text{m} + 0.07\text{m})}} \approx 7.2\text{A}
$$

Given a magnetic field of $50\mu\text{T}$ in the same plane as the magnetotorquer. This produces the maximum torque:

$$
\tau_{max} = A I_{tot} B = (0.07\text{m})^2 \cdot 7.2\text{A} \cdot 50\mu\text{T} \approx 1.75\mu\text{Nm}
$$

The cross area of the wire as a function of effective area, battery voltage and coil power can be found by combining (A.4) with $A_{eff,coil} = n A_{wire}$ and $U^2 = PR$:

$$
A_{wire} = \frac{\sqrt{2 P_{coil} r_s A_{eff,coil} (x_t + y_t)}}{U_{bat}}
$$

For the above given values the cross area is:

$$
\begin{aligned}
A_{wire} &= \frac{\sqrt{2 \cdot 0.05\text{W} \cdot 1.724 \cdot 10^{-8}\Omega\text{m} \cdot 5.00 \cdot 10^{-6}\text{m}^2 \cdot (0.07\text{m} + 0.07\text{m})}}{4.8\text{V}} \\
&= 7.2 \cdot 10^{-9}\text{m}^2
\end{aligned}
$$

Leading to the following wire diameter and number of tunrs:

$$
\begin{aligned}
D_{wire} &= 2 \cdot \sqrt{\frac{A_{wire}}{\pi}} = 2 \cdot \sqrt{\frac{7.2 \cdot 10^{-9}\text{m}^2}{\pi}} = 95\mu\text{m} \\
A_{wire} n &= A_{eff.coil} \Leftrightarrow n = \frac{A_{eff.coil}}{A_{wire}} = \frac{5.00 \cdot 10^{-6}}{7.2 \cdot 10^{-9}\text{m}^2} = 694 \text{ turns}
\end{aligned}
$$